

parametric oscillator

Gonzalo Izús, Marco Santagiustina, Maxi San Miguel, and Pere Colet

Instituto Mediterráneo de Estudios Avanzados, IMEDEA (CSIC-UIB)⁺,

E-07071 Palma de Mallorca, Spain

Abstract

We show the relevance of walk-off effects in pattern formation in a type-II optical parametric oscillator at frequency degeneracy. Neglecting walk-off, only phase patterns are formed and the intensity distribution is homogeneous. Walk-off changes the instability from absolute to convective for some parameter range. In the absolutely unstable regime, it induces for each polarization component of light a competition between two phase stripe patterns (traveling waves) with different wavelength. Phase stripe patterns at each of the wavelengths are equally likely to be selected and after a transient regime of coexistence, one of them takes over. In the convectively unstable regime we show the existence of intensity patterns sustained by noise. The patterns arise from the interference between traveling waves which are excited by noise and dynamically amplified.

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Pattern formation in many nonlinear optical systems has been an active field of investigation¹. New experiments on spatial modulational instability in bulk quadratic media² clearly indicate that pattern formation in optical parametric oscillators (OPO's), theoretically predicted³, could be experimentally observed. Beyond pattern formation and transverse effects in general⁴, other interesting applications of these devices include the generation of squeezed and other non-classical states of the light for low-noise measurements and detection⁵ and might include the storage of information in 2-dimensional optical localized structures^{6,7}. Finally, the interplay among quantum and macroscopic effects in optical systems was also investigated in OPO's¹ which are then a valuable test bed for fundamental physics investigation.

An important characteristic of OPO's is that the crystal birefringence is exploited to phase-match the down-conversion process. The anisotropy can cause the Poynting vectors of the extraordinary polarized components to walk-off the ordinary ones (double-refraction phenomenon) in the plane transverse to the propagation direction. Basically two main situations can be found: in type-I OPO's the twin down-converted photons are polarized in the same direction and always orthogonally to the pump while in type-II OPO's the down-converted beams are polarized along both the ordinary and extraordinary axes, i.e. are polarization non-degenerate. The walk-off effects on the pattern formation processes in type-I OPO's were studied in refs.⁷⁻¹⁰. Pattern formation in a type-II OPO, neglecting the walk-off, has been studied in ref.¹¹ and only phase patterns were found in each polarization component. Here, we study the effects introduced by the walk-off in the pattern formation in a type-II OPO. Two main new phenomena are found for each component of the polarization: a) the competition and pattern selection between two equally preferred traveling waves of

different wavelength; b) in the convective regime, described below, the existence of stripe intensity patterns, induced by the walk-off and sustained by noise.

The equations describing the time evolution of the normalized slowly varying envelopes of the transverse second harmonic (SH) $A_0(\vec{r}, t)$ and the fundamental harmonics (FHs) $A_{1,2}(\vec{r}, t)$ electric fields for a type-II, phase-matched, ring, frequency non-degenerate OPO (NDOPO) in the mean-field approximation can be easily derived as an extension of those of ref.⁸:

$$\partial_t A_0 = \gamma_0 [-(1 + i\Delta_0)A_0 + E_0 + ia_0 \nabla^2 A_0 + 2iK_0 A_1 A_2] + \sqrt{\epsilon_0} \xi_0(\vec{r}, t) \quad (1)$$

$$\partial_t A_1 = \gamma_1 [-(1 + i\Delta_1)A_1 + ia_1 \nabla^2 A_1 + iK_0 A_2^* A_0] + \sqrt{\epsilon_1} \xi_1(\vec{r}, t) \quad (2)$$

$$\partial_t A_2 = \gamma_2 [-(1 + i\Delta_2)A_2 + \rho \partial_y A_2 + ia_2 \nabla^2 A_2 + iK_0 A_1^* A_0] + \sqrt{\epsilon_2} \xi_2(\vec{r}, t) \quad (3)$$

where $A_{0,1}$ are ordinary polarized beams and A_2 is extraordinary polarized; $\vec{r} = (x, y)$ is the transversal spatial vector, t is the time and γ_j , Δ_j and a_j are respectively the cavity decay rate, detuning and diffraction for the field A_j . The nonlinear coefficient is K_0 , E_0 is the injected pump and ρ is the walk-off coefficient. Finally, the last term of each equation is an independent complex Gaussian white noise with zero mean value and delta-correlated in space and time. These terms describe quantum noise in the Wigner representation for the linearized version of eqs. (1-3)¹², but they can account for thermal or input fluctuations as well. In ref.¹¹ similar equations for generic NDOPO have been considered but no walk-off or noise were taken into account while in ref.¹⁰ the study of the walk-off was restricted to transverse perturbation dynamics effects in a Type-I NDOPO and in a parameter region where no pattern is expected when walk-off is neglected.

Eqs. (1-3) have the uniform steady state solution $A_0 = E_0/(1 + i\Delta_0)$, $A_1 = A_2 = 0$. A linear stability analysis is easily performed for the complex vector (A_0, A_1, A_2^*) . The field

A_0 is always stable while A_1 and A_2^* become unstable if the normalized pump intensity $F = K_0 E_0 / (1 + i\Delta_0)$ is such that:

$$|F|^2 \geq 1 + \left[\frac{q^2 \tilde{a} + \tilde{\Delta} + \gamma_2 \rho q_y}{\gamma_1 + \gamma_2} \right]^2 \quad (4)$$

The critical threshold value $|F_c| = 1$ corresponds to traveling waves $A_1, A_2^* \simeq \exp[i\vec{q}\vec{r} + \lambda(\vec{q})t]$, whose two-dimensional real wave-vector \vec{q} belongs to a circle centered at $\vec{q}_0 = (0, q_{0,y})$ with radius $R = |\vec{q} - \vec{q}_0| = \sqrt{q_{0,y}^2 - \tilde{\Delta}/\tilde{a}}$, where $q_{0,y} = -\gamma_2 \rho / 2\tilde{a}$, $\tilde{a} = \gamma_1 a_1 + \gamma_2 a_2$, and $\tilde{\Delta} = \gamma_1 \Delta_1 + \gamma_2 \Delta_2$. In fig. 1 the rings of the most unstable modes for A_1 and A_2 at $|F_c| = 1$ are shown. Without walk-off rings would overlap and, for $|F| > |F_c|$, any \vec{q} mode on the ring, and the opposite for the orthogonal component, can be selected by spontaneous symmetry breaking. Hence a phase pattern appears in A_1 and A_2 while the intensity in both polarizations remains homogeneous¹¹.

As a result of the linear stability analysis, we can highlight several new effects induced by the walk-off. Pattern formation requires $R > 0$ which only occurs for $\tilde{\Delta} < 0$ when $\rho = 0$. The walk-off allows pattern formation even for positive detunings, as it also happens for a type I OPO⁷. The walk-off shifts the center of the rings along the direction of the advection term in opposite directions for A_1 and A_2 and also increases their radius R . Note that, although the walk-off is present only in one polarization component, the shift appears on both. This stems from the conservation of photon momentum which has no transverse component for the input SH. Finally, the walk-off induces the pattern to drift and breaks the rotational symmetry so that the selected wave-vector orientation is parallel to the walk-off direction.

The transversal drift induced by the walk-off can overcome the spreading velocity of

the unstable modes that form the pattern, so that the pattern is effectively washed out of the system. As in other nonlinear optical systems^{8,10,13} this is the convectively unstable regime, which is found for pump amplitudes up to the absolute instability threshold $|F_a|$. This threshold can be determined by extending the linear analysis to complex wave-vectors \vec{k} ^{8,13,14} (formally equivalent to the substitution $i\vec{q} \rightarrow \vec{k}$). Hereafter, we particularize our analysis to the frequency degenerate type-II OPO (DOPO) by setting $\gamma_{1,2} = \gamma$, $\Delta_{1,2} = \Delta$ and $a_{1,2} = a$. We first determine the wave-vector of the most unstable modes by calculating the complex saddle point $\vec{k}_s = (k_x^s, k_y^s)$.

$$\nabla_{\vec{k}} \lambda(\vec{k})|_{\vec{k}_s} = 0, \quad \text{Re}[\nabla_{\vec{k}}^2 \lambda(\vec{k})|_{\vec{k}_s}] > 0 \quad (5)$$

where $\lambda(\vec{k})$ is the eigenvalue which results from the linearization of eqs. (1-3) around the steady-state, i.e.

$$\lambda(\vec{k}) = 2\gamma \left[-1 + \frac{\rho k_y}{2} + \sqrt{|F|^2 - a^2 \left[R^2 + k_x^2 + \left(k_y + i\frac{\rho}{4a} \right)^2 \right]^2} \right] \quad (6)$$

Eqs. (5) have been numerically solved for a complex \vec{k}_s together with the equation $\text{Re}[\lambda(\vec{k}_s)] = 0$ to calculate $|F_a|$. In Fig. 1 we show the absolute instability threshold $|F_a|$ as a function of the detuning Δ for several values of ρ . There are two modes for which A_1 and A_2^* become absolutely unstable at $|F| = |F_a|$, both have the same value $k_x^s = 0$ and different values for k_y^s . These correspond to two equally likely to be selected phase patterns with different wavelength and oriented perpendicularly to the y -axis. The wavelength of each pattern can be calculated along the line of ref.⁷. However, for negative detunings, these wavelengths are very close to that of the modes on the ring in the inset of fig. 1. In the far field, there are only two possible selections of twin beams which conserve momentum, one indicated by the pair of diamonds and the other by the pair of squares.

For large enough systems, and starting from random initial conditions there is a transient regime in which a competition between the two absolutely unstable wave-vectors takes place as shown in fig. 2a. The $Re(A_1)$ shows stripes perpendicular to the drift direction while the intensity $|A_1|^2$ is homogeneous in each of the regions dominated by a given wavelength. The front between these regions displays a set of vertically ordered defects (fig. 2b). The field A_2^* displays a pattern identical to that shown for A_1 . For long time one of the two wave-vectors is selected, as shown in fig. 2c and 2d.

In the convectively unstable regime noise-sustained structures are expected. These structures are locally sustained because, although advected away, they are continuously regenerated by noise which at each space point excites all the unstable modes of each polarization⁸. In particular the most rapidly spreading modes can interfere generating intensity stripes as shown in the numerical solution of fig. 3a. We remark that the existence of patterns in the intensity is a completely new feature for a type-II DOPO, due to the interplay of the convective nature of the instability (i.e. the walk-off) and the presence of noise. At the bottom of the pattern, the generation from noise of randomly oriented stripes can be appreciated. This is a typical feature of noise-sustained structures in two transverse dimensions⁸. These spatial modes are clearly visible in the far field, displayed in Fig. 3b.

In conclusion, in the absolutely unstable regime of a type-II DOPO phase stripes have been observed, the wavelength being randomly selected out of two possible different values. The difference in the wave-vector stems directly from the walk-off. An initial regime of wave-vector competition is also observed. In the convectively unstable regime it has been demonstrated that noise-sustained structures of intensity can be observed. In such a regime noise forces continuously the most unstable traveling waves in both polarization components

and thus creates a structure in the intensity by interference like in a type-I interaction.

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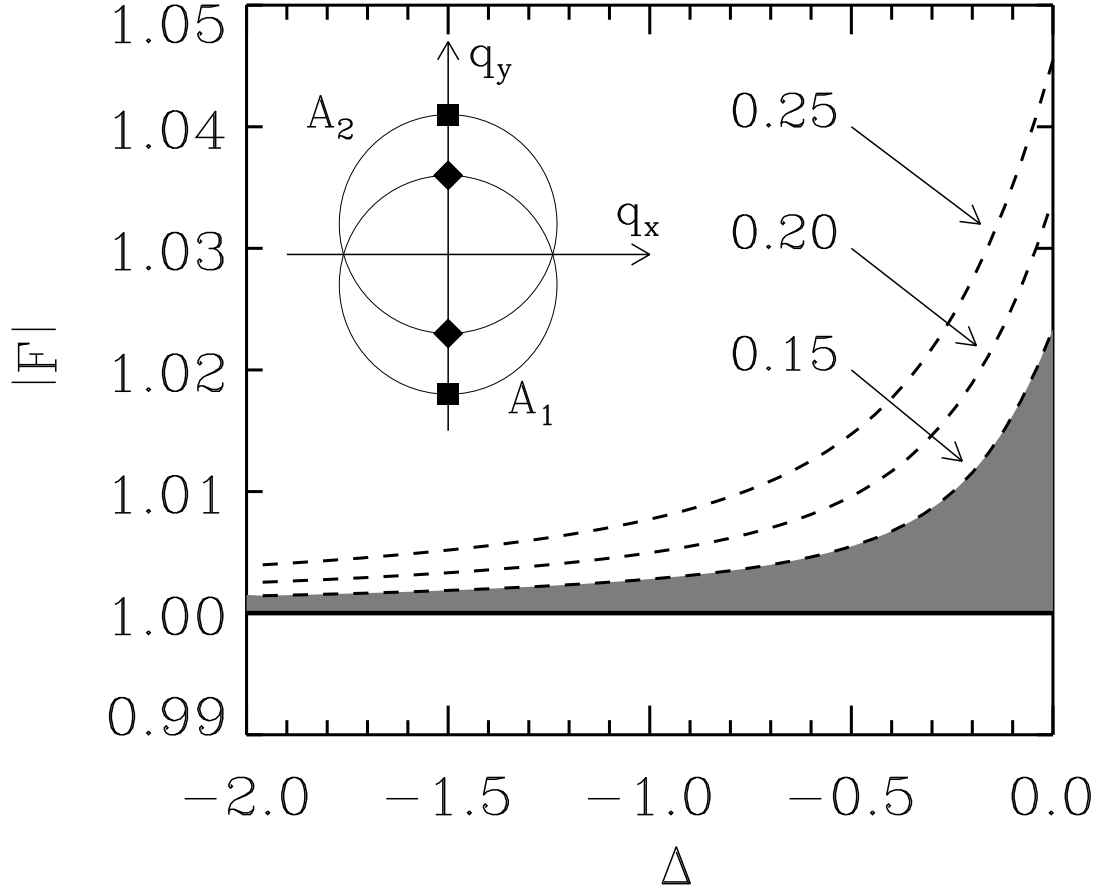


Fig. 1. Absolute threshold $|F_a|$ vs detuning (dashed curves) for several values of ρ . The convective threshold $|F_c|$ is the solid curve. The shadowed region is the convective unstable regime for $\rho = 0.15$. In the inset we show the most unstable modes for both polarization components.

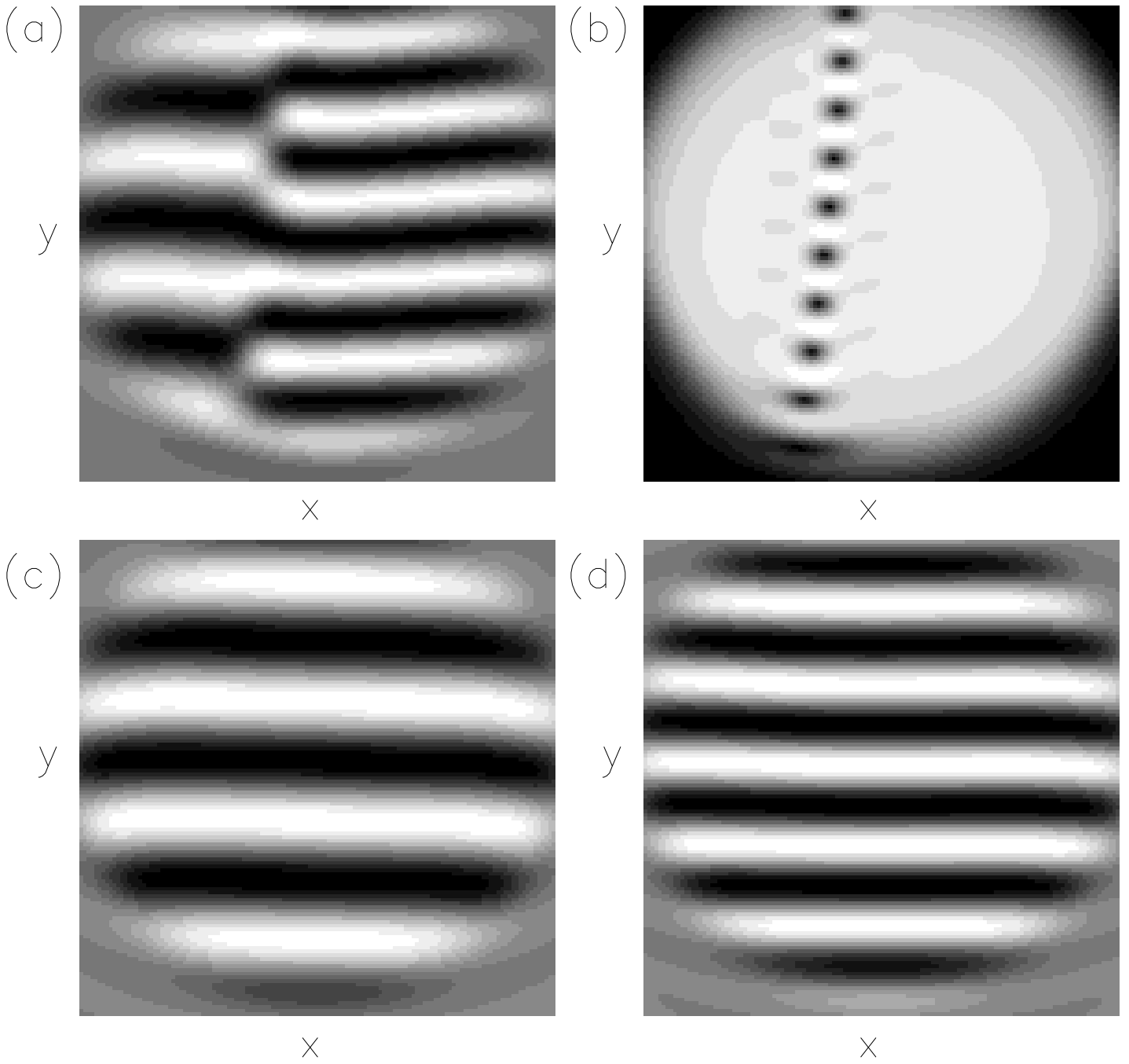


Fig. 2. Transient competition between the most unstable modes a) $Re(A_1)$, b) $|A_1|^2$; c) and d) show $Re(A_1)$ for the final state where the largest or shortest wavelength structure has been selected. Parameters are $2a_0 = a = 0.25, \gamma = 1, \Delta_0 = 0, \Delta = -0.15, K_0 = 1, \rho = 0.15$; E_0 is a super-Gaussian beam of maximum amplitude $E_{max} = 1.05$.

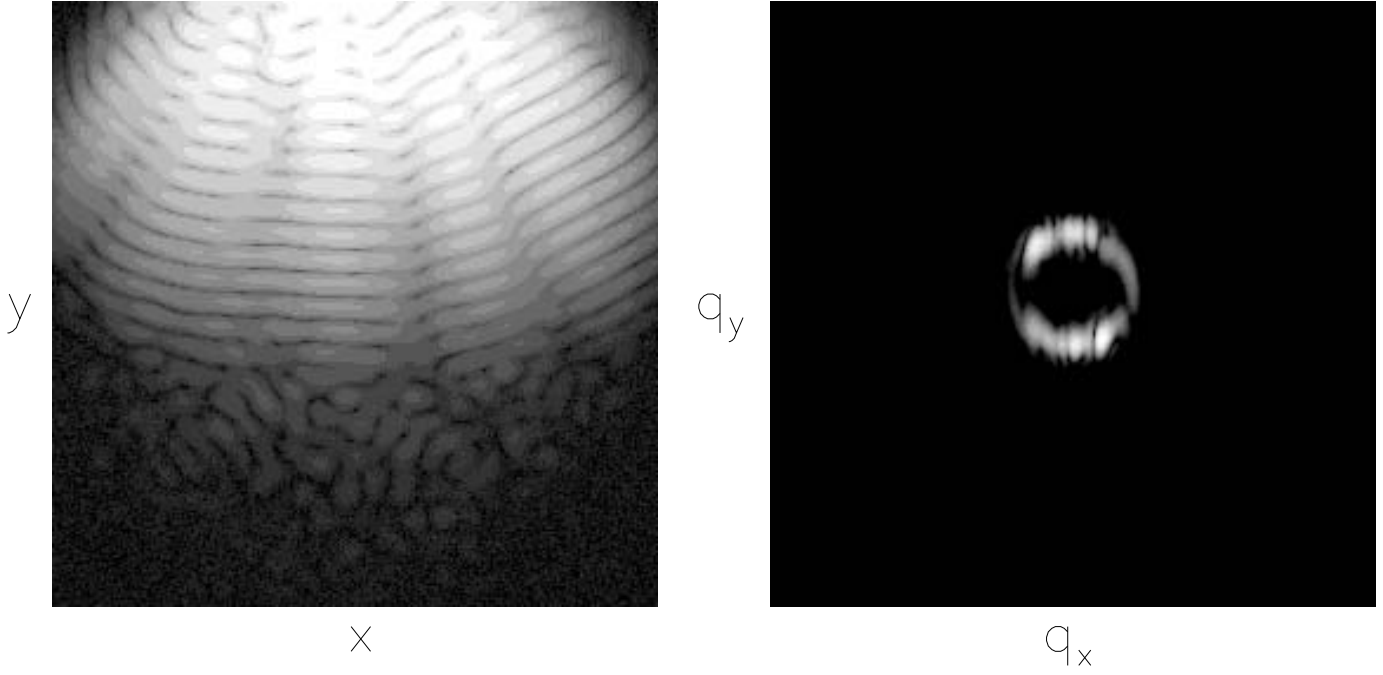


Fig. 3. Intensity of polarization component A_1 in the a) near field and b) far field in the convectively unstable regime. Parameters as in fig. 2 except for $\epsilon_0 = \epsilon_1 = \epsilon_2 = 2 \times 10^{-13}$, $\Delta = -0.2$ and $E_{max} = 1.009$.